Algebraic Logic

Equational Logic A=B

Logical expressions joined by 'equals'.

Axioms of Equations

1. Equality (and Truth) is preserved
   whenever an expression is substituted for its equal.

   `If A = B and B = C, then A = C`

2. Functions of equals are equal.

   `If A = B, then F(A) = F(B).`

Axioms of Equals

1. *Identity*: `A = A`
2. *Commutative*: `A = B iff B = A`
3. *Transitive*: `if A = B and B = C, then A = C`

Axioms of Substitution

0. A[X/Y] means "substitute Y for every X in A"

1. Substituting one expression for another in an equation
   preserves the equality.

   `If A = B, then A[C/E] = B[C/E]`

2. Substituting equal expressions for any subexpressions in an expression preserves the equality.

   `If A = B, then C[A/E] = C[B/E]`

Rule of Standardization

A = B iff (A iff B) = True

Algebraic Proof Techniques

Standard Form:

```
A = B     iff    ((A -> B) and (B -> A))  =  True
```

Direct Transformation:

```
A = B     iff     A => B   or   B => A
```

Mutual Transformation:

```
A = B     iff     A => C   and    B  =>  C
```

Case Analysis:

```
A = B     iff           A[T/E]  =  B[T/E]
                  and   A[F/E]  =  B[F/E]
```

Linear Algebra:

```
A = B     iff          A => T  and  B => T
                  or   A => F  and  B => F
```

Lattice Theory

Lattice theory is the study of a single binary relation ≥
to be read as "is contained in".

A *lattice* is a partially-ordered set (poset), and two elements of which have a **greater lower bound** (glb, meet) and a **least upper bound** (lub, join)

A *boolean lattice* is a complemented, distributed lattice, and forms a boolean algebra.